Complete Solutions Manual

Linear Algebra
A Modern Introduction

FOURTH EDITION

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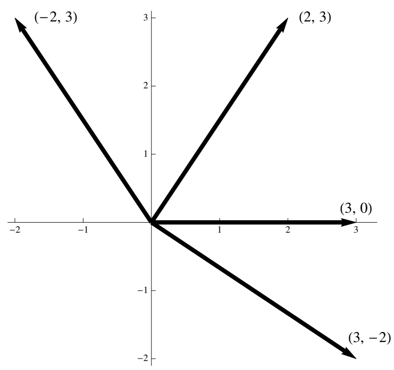
Chapter 1

Vectors

1.1 The Geometry and Algebra of Vectors

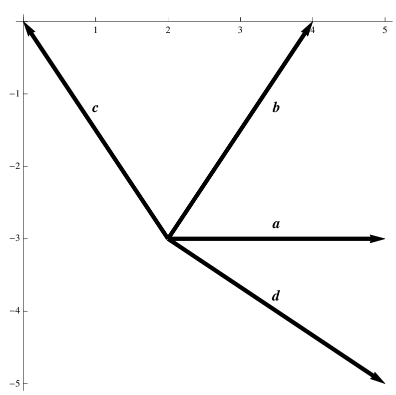
1.



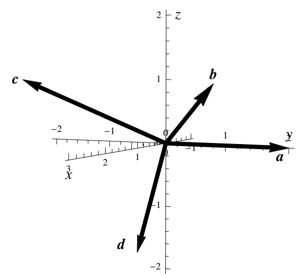
2. Since

$$\begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \qquad \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix},$$

plotting those vectors gives



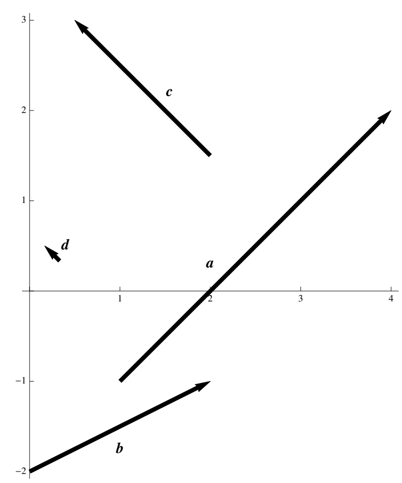
3.



4. Since the heads are all at $(3, 2, 1)$, the tails are at

$$\begin{bmatrix}3\\2\\1\end{bmatrix} - \begin{bmatrix}0\\2\\0\end{bmatrix} = \begin{bmatrix}3\\0\\1\end{bmatrix}, \quad \begin{bmatrix}3\\2\\1\end{bmatrix} - \begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}, \quad \begin{bmatrix}3\\2\\1\end{bmatrix} - \begin{bmatrix}1\\-2\\1\end{bmatrix} = \begin{bmatrix}2\\4\\0\end{bmatrix}, \quad \begin{bmatrix}3\\2\\1\end{bmatrix} - \begin{bmatrix}-1\\-1\\-2\end{bmatrix} = \begin{bmatrix}4\\3\\3\end{bmatrix}.$$

5. The four vectors $\overrightarrow{AB}$ are



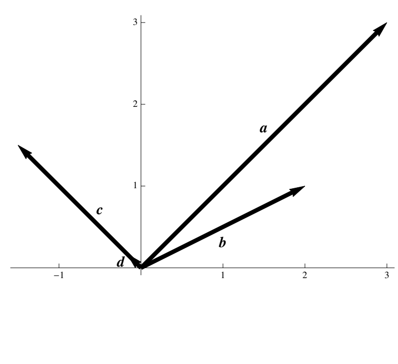
In standard position, the vectors are

(a) $\overrightarrow{AB} = [4 - 1, 2 - (-1)] = [3, 3]$.

(b) $\overrightarrow{AB} = [2 - 0, -1 - (-2)] = [2, 1]$

(c) $\overrightarrow{AB} = \left[\frac{1}{2} - 2, 3 - \frac{3}{2}\right] = \left[-\frac{3}{2}, \frac{3}{2}\right]$

(d) $\overrightarrow{AB} = \left[\frac{1}{6} - \frac{1}{3}, \frac{1}{2} - \frac{1}{3}\right] = \left[-\frac{1}{6}, \frac{1}{6}\right]$.



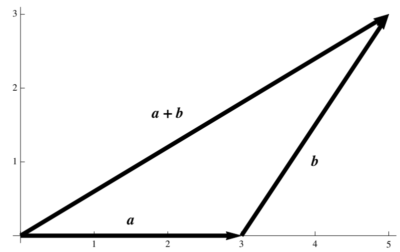
6. Recall the notation that $[a, b]$ denotes a move of a units horizontally and b units vertically. Then during the first part of the walk, the hiker walks 4 km north, so $\mathbf{a} = [0, 4]$. During the second part of the walk, the hiker walks a distance of 5 km northeast. From the components, we get

$$\mathbf{b} = [5\cos 45^\circ,\ 5\sin 45^\circ] = \left[\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right].$$

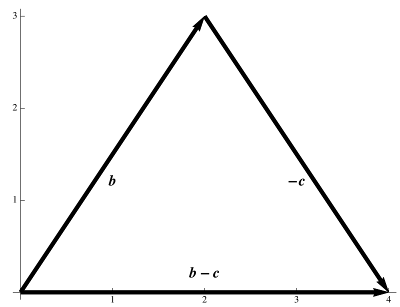
Thus the net displacement vector is

$$\mathbf{c} = \mathbf{a} + \mathbf{b} = \left[\frac{5\sqrt{2}}{2}, 4 + \frac{5\sqrt{2}}{2}\right].$$

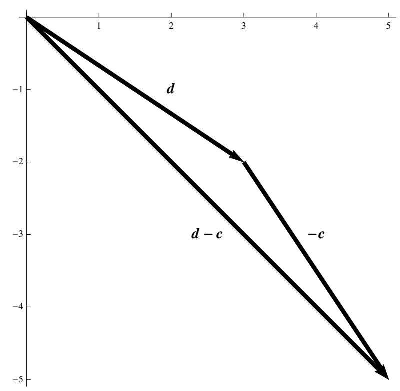
7. $\mathbf{a} + \mathbf{b} = \begin{bmatrix}3\\0\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}3+2\\0+3\end{bmatrix} = \begin{bmatrix}5\\3\end{bmatrix}.$



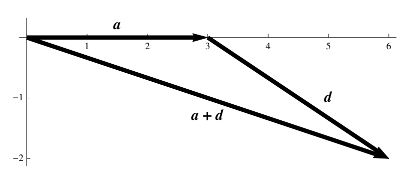
8. $\mathbf{b} - \mathbf{c} = \begin{bmatrix}2\\3\end{bmatrix} - \begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}2-(-2)\\3-3\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix}.$



9. $\mathbf{d} - \mathbf{c} = \begin{bmatrix}3\\-2\end{bmatrix} - \begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}5\\-5\end{bmatrix}.$



10. $\mathbf{a} + \mathbf{d} = \begin{bmatrix}3\\0\end{bmatrix} + \begin{bmatrix}3\\-2\end{bmatrix} = \begin{bmatrix}3+3\\0+(-2)\end{bmatrix} = \begin{bmatrix}6\\-2\end{bmatrix}.$



11. $2\mathbf{a} + 3\mathbf{c} = 2[0, 2, 0] + 3[1, -2, 1] = [2\cdot 0, 2\cdot 2, 2\cdot 0] + [3\cdot 1, 3\cdot(-2), 3\cdot 1] = [3, -2, 3].$

12.

$$\begin{aligned}3\mathbf{b} - 2\mathbf{c} + \mathbf{d} &= 3[3, 2, 1] - 2[1, -2, 1] + [-1, -1, -2]\\ &= [3\cdot 3, 3\cdot 2, 3\cdot 1] + [-2\cdot 1, -2\cdot(-2), -2\cdot 1] + [-1, -1, -2]\\ &= [6, 9, -1].\end{aligned}$$

13. $\mathbf{u} = [\cos 60°, \sin 60°] = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$, and $\mathbf{v} = [\cos 210°, \sin 210°] = \left[-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right]$, so that

$$\mathbf{u}+\mathbf{v} = \left[\frac{1}{2} - \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} - \frac{1}{2}\right], \quad \mathbf{u}-\mathbf{v} = \left[\frac{1}{2} + \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} + \frac{1}{2}\right].$$

14. **(a)** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

(b) Since $\overrightarrow{OC} = \overrightarrow{AB}$, we have $\overrightarrow{BC} = \overrightarrow{OC} - \mathbf{b} = (\mathbf{b} - \mathbf{a}) - \mathbf{b} = -\mathbf{a}$.

(c) $\overrightarrow{AD} = -2\mathbf{a}$.

(d) $\overrightarrow{CF} = -2\overrightarrow{OC} = -2\overrightarrow{AB} = -2(\mathbf{b} - \mathbf{a}) = 2(\mathbf{a} - \mathbf{b})$.

(e) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (\mathbf{b} - \mathbf{a}) + (-\mathbf{a}) = \mathbf{b} - 2\mathbf{a}$.

(f) Note that $\overrightarrow{FA}$ and $\overrightarrow{OB}$ are equal, and that $\overrightarrow{DE} = -\overrightarrow{AB}$. Then

$$\overrightarrow{BC} + \overrightarrow{DE} + \overrightarrow{FA} = -\mathbf{a} - \overrightarrow{AB} + \overrightarrow{OB} = -\mathbf{a} - (\mathbf{b} - \mathbf{a}) + \mathbf{b} = \mathbf{0}.$$

15. $2(\mathbf{a} - 3\mathbf{b}) + 3(2\mathbf{b} + \mathbf{a}) \overset{\substack{\text{property e.}\\ \text{distributivity}}}{=} (2\mathbf{a} - 6\mathbf{b}) + (6\mathbf{b} + 3\mathbf{a}) \overset{\substack{\text{property b.}\\ \text{associativity}}}{=} (2\mathbf{a} + 3\mathbf{a}) + (-6\mathbf{b} + 6\mathbf{b}) = 5\mathbf{a}$.

16.

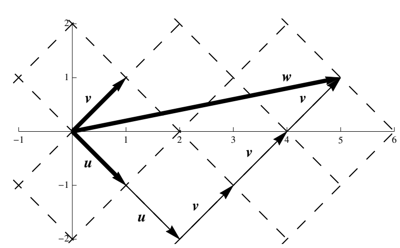
$$\begin{aligned} -3(\mathbf{a} - \mathbf{c}) + 2(\mathbf{a} + 2\mathbf{b}) + 3(\mathbf{c} - \mathbf{b}) &\overset{\substack{\text{property e.}\\ \text{distributivity}}}{=} (-3\mathbf{a} + 3\mathbf{c}) + (2\mathbf{a} + 4\mathbf{b}) + (3\mathbf{c} - 3\mathbf{b}) \\ &\overset{\substack{\text{property b.}\\ \text{associativity}}}{=} (-3\mathbf{a} + 2\mathbf{a}) + (4\mathbf{b} - 3\mathbf{b}) + (3\mathbf{c} + 3\mathbf{c}) \\ &= -\mathbf{a} + \mathbf{b} + 6\mathbf{c}. \end{aligned}$$

17. $\mathbf{x} - \mathbf{a} = 2(\mathbf{x} - 2\mathbf{a}) = 2\mathbf{x} - 4\mathbf{a} \Rightarrow \mathbf{x} - 2\mathbf{x} = \mathbf{a} - 4\mathbf{a} \Rightarrow -\mathbf{x} = -3\mathbf{a} \Rightarrow \mathbf{x} = 3\mathbf{a}$.

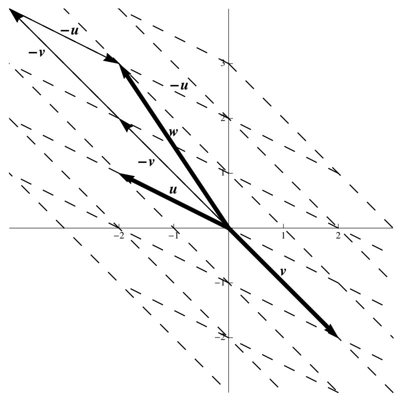
18.

$$\begin{aligned} \mathbf{x} + 2\mathbf{a} - \mathbf{b} = 3(\mathbf{x} + \mathbf{a}) - 2(2\mathbf{a} - \mathbf{b}) = 3\mathbf{x} + 3\mathbf{a} - 4\mathbf{a} + 2\mathbf{b} \quad &\Rightarrow \\ \mathbf{x} - 3\mathbf{x} = -\mathbf{a} - 2\mathbf{a} + 2\mathbf{b} + \mathbf{b} \quad &\Rightarrow \\ -2\mathbf{x} = -3\mathbf{a} + 3\mathbf{b} \quad &\Rightarrow \\ \mathbf{x} = \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}. \end{aligned}$$

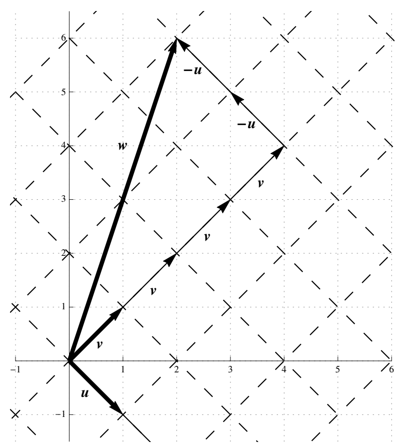
19. We have $2\mathbf{u} + 3\mathbf{v} = 2[1, -1] + 3[1, 1] = [2 \cdot 1 + 3 \cdot 1, 2 \cdot (-1) + 3 \cdot 1] = [5, 1]$. Plots of all three vectors are



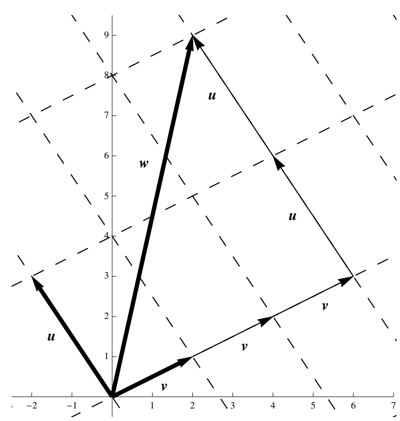
20. We have $-\mathbf{u} - 2\mathbf{v} = -[-2, 1] - 2[2, -2] = [-(-2) - 2 \cdot 2, -1 - 2 \cdot (-2)] = [-2, 3]$. Plots of all three vectors are



21. From the diagram, we see that $\mathbf{w} = -2\mathbf{u} + 4\mathbf{v}$.

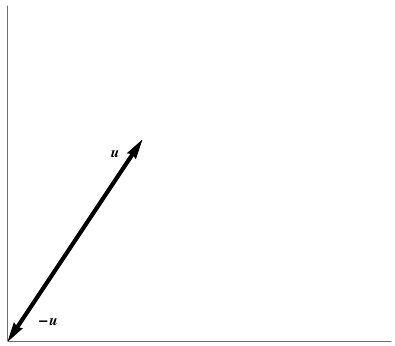


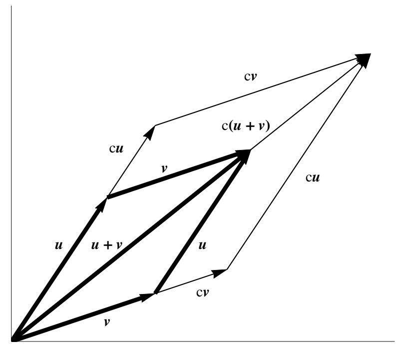
22. From the diagram, we see that $\mathbf{w} = 2\mathbf{u} + 3\mathbf{v}$.



23. Property (d) states that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$. The first diagram below shows $\mathbf{u}$ along with $-\mathbf{u}$. Then, as the diagonal of the parallelogram, the resultant vector is $\mathbf{0}$.

Property (e) states that $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$. The second figure illustrates this.





24. Let $\mathbf{u} = [u_1, u_2, \ldots, u_n]$ and $\mathbf{v} = [v_1, v_2, \ldots, v_n]$, and let c and d be scalars in $\mathbb{R}$.
Property (d):

$$\begin{aligned}\mathbf{u} + (-\mathbf{u}) &= [u_1, u_2, \ldots, u_n] + (-1[u_1, u_2, \ldots, u_n]) \\ &= [u_1, u_2, \ldots, u_n] + [-u_1, -u_2, \ldots, -u_n] \\ &= [u_1 - u_1, u_2 - u_2, \ldots, u_n - u_n] \\ &= [0, 0, \ldots, 0] = \mathbf{0}.\end{aligned}$$

Property (e):

$$\begin{aligned}c(\mathbf{u} + \mathbf{v}) &= c\left([u_1, u_2, \ldots, u_n] + [v_1, v_2, \ldots, v_n]\right) \\ &= c\left([u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n]\right) \\ &= [c(u_1 + v_1), c(u_2 + v_2), \ldots, c(u_n + v_n)] \\ &= [cu_1 + cv_1, cu_2 + cv_2, \ldots, cu_n + cv_n] \\ &= [cu_1, cu_2, \ldots, cu_n] + [cv_1, cv_2, \ldots, cv_n] \\ &= c[u_1, u_2, \ldots, u_n] + c[v_1, v_2, \ldots, v_n] \\ &= c\mathbf{u} + c\mathbf{v}.\end{aligned}$$

Property (f):

$$\begin{aligned}(c + d)\mathbf{u} &= (c + d)[u_1, u_2, \ldots, u_n] \\ &= [(c + d)u_1, (c + d)u_2, \ldots, (c + d)u_n] \\ &= [cu_1 + du_1, cu_2 + du_2, \ldots, cu_n + du_n] \\ &= [cu_1, cu_2, \ldots, cu_n] + [du_1, du_2, \ldots, du_n] \\ &= c[u_1, u_2, \ldots, u_n] + d[u_1, u_2, \ldots, u_n] \\ &= c\mathbf{u} + d\mathbf{u}.\end{aligned}$$

Property (g):

$$\begin{aligned}c(d\mathbf{u}) &= c(d[u_1, u_2, \ldots, u_n]) \\ &= c[du_1, du_2, \ldots, du_n] \\ &= [cdu_1, cdu_2, \ldots, cdu_n] \\ &= [(cd)u_1, (cd)u_2, \ldots, (cd)u_n] \\ &= (cd)[u_1, u_2, \ldots, u_n] \\ &= (cd)\mathbf{u}.\end{aligned}$$

25. $\mathbf{u} + \mathbf{v} = [0, 1] + [1, 1] = [1, 0]$.

26. $\mathbf{u} + \mathbf{v} = [1, 1, 0] + [1, 1, 1] = [0, 0, 1]$.

27. $\mathbf{u}+\mathbf{v} = [1,0,1,1]+[1,1,1,1] = [0,1,0,0]$.

28. $\mathbf{u}+\mathbf{v} = [1,1,0,1,0]+[0,1,1,1,0] = [1,0,1,0,0]$.

29.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

30.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

·	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

31. $2+2+2=6=0$ in $\mathbb{Z}_3$.

32. $2\cdot 2\cdot 2=3\cdot 2=0$ in $\mathbb{Z}_3$.

33. $2(2+1+2)=2\cdot 2=3\cdot 1+1=1$ in $\mathbb{Z}_3$.

34. $3+1+2+3=4\cdot 2+1=1$ in $\mathbb{Z}_4$.

35. $2\cdot 3\cdot 2=4\cdot 3+0=0$ in $\mathbb{Z}_4$.

36. $3(3+3+2)=4\cdot 6+0=0$ in $\mathbb{Z}_4$.

37. $2+1+2+2+1=2$ in $\mathbb{Z}_3$, $2+1+2+2+1=0$ in $\mathbb{Z}_4$, $2+1+2+2+1=3$ in $\mathbb{Z}_5$.

38. $(3+4)(3+2+4+2)=2\cdot 1=2$ in $\mathbb{Z}_5$.

39. $8(6+4+3)=8\cdot 4=5$ in $\mathbb{Z}_9$.

40. $2^{100}=\left(2^{10}\right)^{10}=(1024)^{10}=1^{10}=1$ in $\mathbb{Z}_{11}$.

41. $[2,1,2]+[2,0,1]=[1,1,0]$ in $\mathbb{Z}_3^3$.

42. $2[2,\ 2,\ 1]=[2\cdot 2,\ 2\cdot 2,\ 2\cdot 1]=[1,\ 1,\ 2]$ in $\mathbb{Z}_3^3$.

43. $2([3,1,1,2]+[3,3,2,1])=2[2,0,3,3]=[2\cdot 2,\ 2\cdot 0,\ 2\cdot 3,\ 2\cdot 3]=[0,0,2,2]$ in $\mathbb{Z}_4^4$.
$2([3,1,1,2]+[3,3,2,1])=2[1,4,3,3]=[2\cdot 1,\ 2\cdot 4,\ 2\cdot 3,\ 2\cdot 3]=[2,3,1,1]$ in $\mathbb{Z}_5^4$.

44. $x=2+(-3)=2+2=4$ in $\mathbb{Z}_5$.

45. $x=1+(-5)=1+1=2$ in $\mathbb{Z}_6$

46. $x=2^{-1}=2$ in $\mathbb{Z}_3$.

47. No solution. 2 times anything is always even, so cannot leave a remainder of 1 when divided by 4.

48. $x=2^{-1}=3$ in $\mathbb{Z}_5$.

49. $x=3^{-1}4=2\cdot 4=3$ in $\mathbb{Z}_5$.

50. No solution. 3 times anything is always a multiple of 3, so it cannot leave a remainder of 4 when divided by 6 (which is also a multiple of 3).

51. No solution. 6 times anything is always even, so it cannot leave an odd number as a remainder when divided by 8.

52. $x = 8^{-1}9 = 7 \cdot 9 = 8$ in $\mathbb{Z}_{11}$

53. $x = 2^{-1}(2 + (-3)) = 3(2 + 2) = 2$ in $\mathbb{Z}_5$.

54. No solution. This equation is the same as $4x = 2 - 5 = -3 = 3$ in $\mathbb{Z}_6$. But 4 times anything is even, so it cannot leave a remainder of 3 when divided by 6 (which is also even).

55. Add 5 to both sides to get $6x = 6$, so that $x = 1$ or $x = 5$ (since $6 \cdot 1 = 6$ and $6 \cdot 5 = 30 = 6$ in $\mathbb{Z}_8$).

56. (a) All values. (b) All values. (c) All values.

57. **(a)** All $a \neq 0$ in $\mathbb{Z}_5$ have a solution because 5 is a prime number.

(b) $a = 1$ and $a = 5$ because they have no common factors with 6 other than 1.

(c) a and m can have no common factors other than 1; that is, the *greatest common divisor*, gcd, of a and m is 1.

1.2 Length and Angle: The Dot Product

1. Following Example 1.15, $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = (-1) \cdot 3 + 2 \cdot 1 = -3 + 2 = -1$.

2. Following Example 1.15, $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 6 \end{bmatrix} = 3 \cdot 4 + (-2) \cdot 6 = 12 - 12 = 0$.

3. $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 1 = 2 + 6 + 3 = 11$.

4. $\mathbf{u} \cdot \mathbf{v} = 3.2 \cdot 1.5 + (-0.6) \cdot 4.1 + (-1.4) \cdot (-0.2) = 4.8 - 2.46 + 0.28 = 2.62$.

5. $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -\sqrt{2} \\ 0 \\ -5 \end{bmatrix} = 1 \cdot 4 + \sqrt{2} \cdot (-\sqrt{2}) + \sqrt{3} \cdot 0 + 0 \cdot (-5) = 4 - 2 = 2$.

6. $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 1.12 \\ -3.25 \\ 2.07 \\ -1.83 \end{bmatrix} \cdot \begin{bmatrix} -2.29 \\ 1.72 \\ 4.33 \\ -1.54 \end{bmatrix} = -1.12 \cdot 2.29 - 3.25 \cdot 1.72 + 2.07 \cdot 4.33 - 1.83 \cdot (-1.54) = 3.6265$.

7. Finding a unit vector $\mathbf{v}$ in the same direction as a given vector $\mathbf{u}$ is called ***normalizing*** the vector $\mathbf{u}$. Proceed as in Example 1.19:

$$\|\mathbf{u}\| = \sqrt{(-1)^2 + 2^2} = \sqrt{5},$$

so a unit vector $\mathbf{v}$ in the same direction as $\mathbf{u}$ is

$$\mathbf{v} = \frac{1}{\|\mathbf{u}\|}\mathbf{u} = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}.$$

8. Proceed as in Example 1.19:

$$\|\mathbf{u}\| = \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13},$$

so a unit vector $\mathbf{v}$ in the direction of $\mathbf{u}$ is

$$\mathbf{v} = \frac{1}{\|\mathbf{u}\|}\mathbf{u} = \frac{1}{\sqrt{13}} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{13}} \\ -\frac{2}{\sqrt{13}} \end{bmatrix}.$$