

Complete Solutions Manual

Linear Algebra
A Modern Introduction

FOURTH EDITION

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Prepared by
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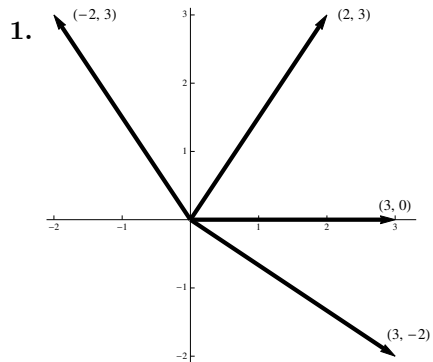
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Chapter 1

Vectors

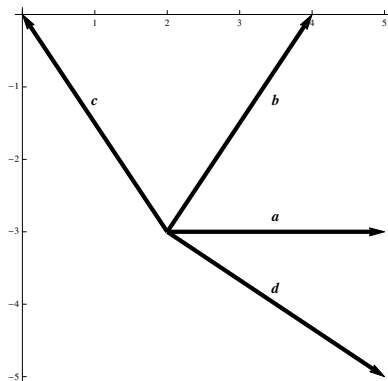
1.1 The Geometry and Algebra of Vectors



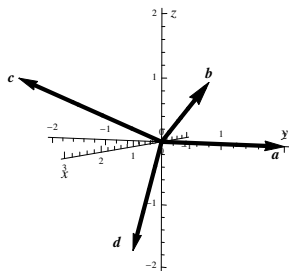
2. Since

$$\begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix},$$

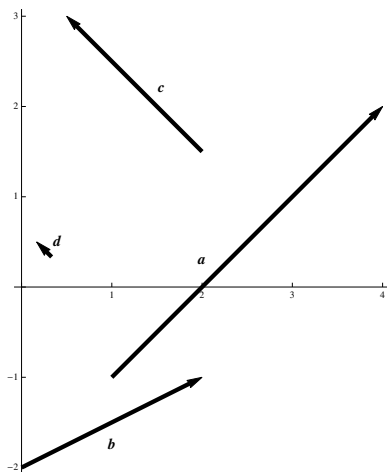
plotting those vectors gives



3.

4. Since the heads are all at $(3, 2, 1)$, the tails are at

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}.$$

5. The four vectors \overrightarrow{AB} are

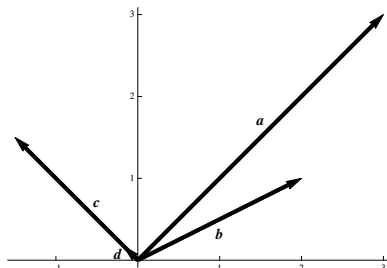
In standard position, the vectors are

$$(a) \overrightarrow{AB} = [4 - 1, 2 - (-1)] = [3, 3].$$

$$(b) \overrightarrow{AB} = [2 - 0, -1 - (-2)] = [2, 1]$$

$$(c) \overrightarrow{AB} = [\frac{1}{2} - 2, 3 - \frac{3}{2}] = [-\frac{3}{2}, \frac{3}{2}]$$

$$(d) \overrightarrow{AB} = [\frac{1}{6} - \frac{1}{3}, \frac{1}{2} - \frac{1}{3}] = [-\frac{1}{6}, \frac{1}{6}].$$



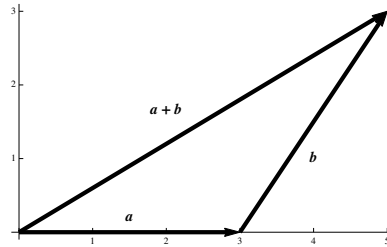
6. Recall the notation that $[a, b]$ denotes a move of a units horizontally and b units vertically. Then during the first part of the walk, the hiker walks 4 km north, so $\mathbf{a} = [0, 4]$. During the second part of the walk, the hiker walks a distance of 5 km northeast. From the components, we get

$$\mathbf{b} = [5 \cos 45^\circ, 5 \sin 45^\circ] = \left[\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right].$$

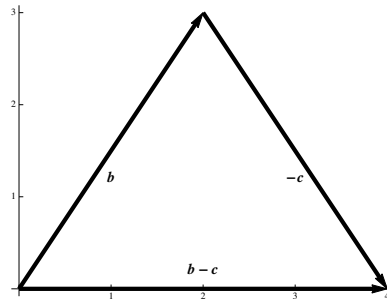
Thus the net displacement vector is

$$\mathbf{c} = \mathbf{a} + \mathbf{b} = \left[\frac{5\sqrt{2}}{2}, 4 + \frac{5\sqrt{2}}{2} \right].$$

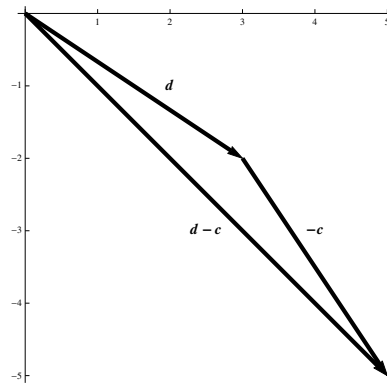
7. $\mathbf{a} + \mathbf{b} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3+2 \\ 0+3 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}.$



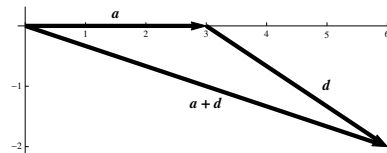
8. $\mathbf{b} - \mathbf{c} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 - (-2) \\ 3 - 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}.$



9. $\mathbf{d} - \mathbf{c} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}.$



10. $\mathbf{a} + \mathbf{d} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3+3 \\ 0+(-2) \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}.$



11. $2\mathbf{a} + 3\mathbf{c} = 2[0, 2, 0] + 3[1, -2, 1] = [2 \cdot 0, 2 \cdot 2, 2 \cdot 0] + [3 \cdot 1, 3 \cdot (-2), 3 \cdot 1] = [3, -2, 3].$

12.

$$\begin{aligned} 3\mathbf{b} - 2\mathbf{c} + \mathbf{d} &= 3[3, 2, 1] - 2[1, -2, 1] + [-1, -1, -2] \\ &= [3 \cdot 3, 3 \cdot 2, 3 \cdot 1] + [-2 \cdot 1, -2 \cdot (-2), -2 \cdot 1] + [-1, -1, -2] \\ &= [6, 9, -1]. \end{aligned}$$

13. $\mathbf{u} = [\cos 60^\circ, \sin 60^\circ] = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$, and $\mathbf{v} = [\cos 210^\circ, \sin 210^\circ] = \left[-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right]$, so that

$$\mathbf{u} + \mathbf{v} = \left[\frac{1}{2} - \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} - \frac{1}{2}\right], \quad \mathbf{u} - \mathbf{v} = \left[\frac{1}{2} + \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} + \frac{1}{2}\right].$$

14. (a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

(b) Since $\overrightarrow{OC} = \overrightarrow{AB}$, we have $\overrightarrow{BC} = \overrightarrow{OC} - \mathbf{b} = (\mathbf{b} - \mathbf{a}) - \mathbf{b} = -\mathbf{a}$.

(c) $\overrightarrow{AD} = -2\mathbf{a}$.

(d) $\overrightarrow{CF} = -2\overrightarrow{OC} = -2\overrightarrow{AB} = -2(\mathbf{b} - \mathbf{a}) = 2(\mathbf{a} - \mathbf{b})$.

(e) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (\mathbf{b} - \mathbf{a}) + (-\mathbf{a}) = \mathbf{b} - 2\mathbf{a}$.

(f) Note that \overrightarrow{FA} and \overrightarrow{OB} are equal, and that $\overrightarrow{DE} = -\overrightarrow{AB}$. Then

$$\overrightarrow{BC} + \overrightarrow{DE} + \overrightarrow{FA} = -\mathbf{a} - \overrightarrow{AB} + \overrightarrow{OB} = -\mathbf{a} - (\mathbf{b} - \mathbf{a}) + \mathbf{b} = \mathbf{0}.$$

15. $2(\mathbf{a} - 3\mathbf{b}) + 3(2\mathbf{b} + \mathbf{a}) \stackrel{\text{property e. distributivity}}{=} (2\mathbf{a} - 6\mathbf{b}) + (6\mathbf{b} + 3\mathbf{a}) \stackrel{\text{property b. associativity}}{=} (2\mathbf{a} + 3\mathbf{a}) + (-6\mathbf{b} + 6\mathbf{b}) = 5\mathbf{a}$.

16.

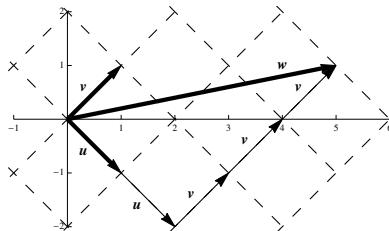
$$\begin{aligned} -3(\mathbf{a} - \mathbf{c}) + 2(\mathbf{a} + 2\mathbf{b}) + 3(\mathbf{c} - \mathbf{b}) &\stackrel{\text{property e. distributivity}}{=} (-3\mathbf{a} + 3\mathbf{c}) + (2\mathbf{a} + 4\mathbf{b}) + (3\mathbf{c} - 3\mathbf{b}) \\ &\stackrel{\text{property b. associativity}}{=} (-3\mathbf{a} + 2\mathbf{a}) + (4\mathbf{b} - 3\mathbf{b}) + (3\mathbf{c} + 3\mathbf{c}) \\ &= -\mathbf{a} + \mathbf{b} + 6\mathbf{c}. \end{aligned}$$

17. $\mathbf{x} - \mathbf{a} = 2(\mathbf{x} - 2\mathbf{a}) = 2\mathbf{x} - 4\mathbf{a} \Rightarrow \mathbf{x} - 2\mathbf{x} = \mathbf{a} - 4\mathbf{a} \Rightarrow -\mathbf{x} = -3\mathbf{a} \Rightarrow \mathbf{x} = 3\mathbf{a}$.

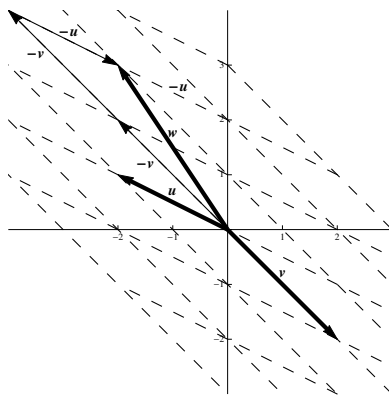
18.

$$\begin{aligned} \mathbf{x} + 2\mathbf{a} - \mathbf{b} &= 3(\mathbf{x} + \mathbf{a}) - 2(2\mathbf{a} - \mathbf{b}) = 3\mathbf{x} + 3\mathbf{a} - 4\mathbf{a} + 2\mathbf{b} \Rightarrow \\ \mathbf{x} - 3\mathbf{x} &= -\mathbf{a} - 2\mathbf{a} + 2\mathbf{b} + \mathbf{b} \Rightarrow \\ -2\mathbf{x} &= -3\mathbf{a} + 3\mathbf{b} \Rightarrow \\ \mathbf{x} &= \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}. \end{aligned}$$

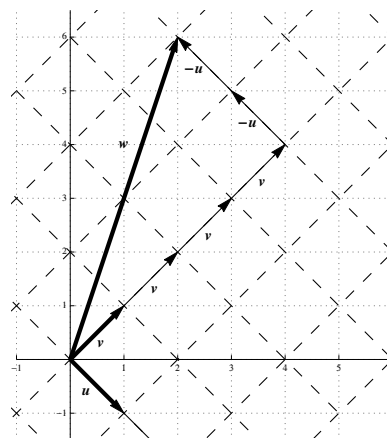
19. We have $2\mathbf{u} + 3\mathbf{v} = 2[1, -1] + 3[1, 1] = [2 \cdot 1 + 3 \cdot 1, 2 \cdot (-1) + 3 \cdot 1] = [5, 1]$. Plots of all three vectors are



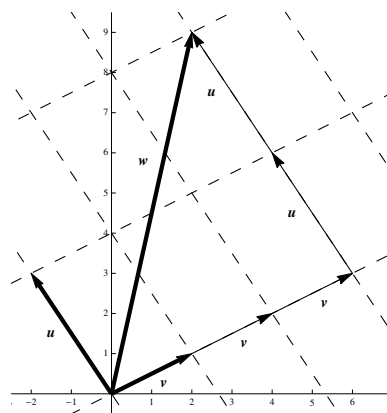
20. We have $-\mathbf{u} - 2\mathbf{v} = -[-2, 1] - 2[2, -2] = [-(-2) - 2 \cdot 2, -1 - 2 \cdot (-2)] = [-2, 3]$. Plots of all three vectors are



21. From the diagram, we see that $\mathbf{w} = -2\mathbf{u} + 4\mathbf{v}$.

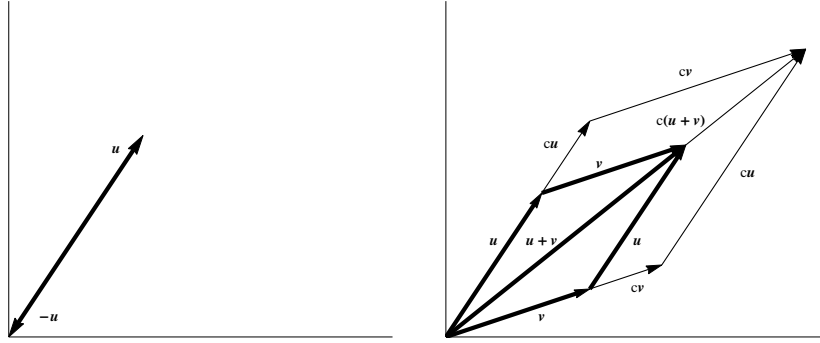


22. From the diagram, we see that $\mathbf{w} = 2\mathbf{u} + 3\mathbf{v}$.



23. Property (d) states that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$. The first diagram below shows \mathbf{u} along with $-\mathbf{u}$. Then, as the diagonal of the parallelogram, the resultant vector is $\mathbf{0}$.

Property (e) states that $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$. The second figure illustrates this.



24. Let $\mathbf{u} = [u_1, u_2, \dots, u_n]$ and $\mathbf{v} = [v_1, v_2, \dots, v_n]$, and let c and d be scalars in \mathbb{R} .

Property (d):

$$\begin{aligned} \mathbf{u} + (-\mathbf{u}) &= [u_1, u_2, \dots, u_n] + (-1[u_1, u_2, \dots, u_n]) \\ &= [u_1, u_2, \dots, u_n] + [-u_1, -u_2, \dots, -u_n] \\ &= [u_1 - u_1, u_2 - u_2, \dots, u_n - u_n] \\ &= [0, 0, \dots, 0] = \mathbf{0}. \end{aligned}$$

Property (e):

$$\begin{aligned} c(\mathbf{u} + \mathbf{v}) &= c([u_1, u_2, \dots, u_n] + [v_1, v_2, \dots, v_n]) \\ &= c([u_1 + v_1, u_2 + v_2, \dots, u_n + v_n]) \\ &= [c(u_1 + v_1), c(u_2 + v_2), \dots, c(u_n + v_n)] \\ &= [cu_1 + cv_1, cu_2 + cv_2, \dots, cu_n + cv_n] \\ &= [cu_1, cu_2, \dots, cu_n] + [cv_1, cv_2, \dots, cv_n] \\ &= c[u_1, u_2, \dots, u_n] + c[v_1, v_2, \dots, v_n] \\ &= c\mathbf{u} + c\mathbf{v}. \end{aligned}$$

Property (f):

$$\begin{aligned} (c + d)\mathbf{u} &= (c + d)[u_1, u_2, \dots, u_n] \\ &= [(c + d)u_1, (c + d)u_2, \dots, (c + d)u_n] \\ &= [cu_1 + du_1, cu_2 + du_2, \dots, cu_n + du_n] \\ &= [cu_1, cu_2, \dots, cu_n] + [du_1, du_2, \dots, du_n] \\ &= c[u_1, u_2, \dots, u_n] + d[u_1, u_2, \dots, u_n] \\ &= c\mathbf{u} + d\mathbf{u}. \end{aligned}$$

Property (g):

$$\begin{aligned} c(d\mathbf{u}) &= c(d[u_1, u_2, \dots, u_n]) \\ &= c[du_1, du_2, \dots, du_n] \\ &= [cdu_1, cdu_2, \dots, cdu_n] \\ &= [(cd)u_1, (cd)u_2, \dots, (cd)u_n] \\ &= (cd)[u_1, u_2, \dots, u_n] \\ &= (cd)\mathbf{u}. \end{aligned}$$

25. $\mathbf{u} + \mathbf{v} = [0, 1] + [1, 1] = [1, 0]$.

26. $\mathbf{u} + \mathbf{v} = [1, 1, 0] + [1, 1, 1] = [0, 0, 1]$.

27. $\mathbf{u} + \mathbf{v} = [1, 0, 1, 1] + [1, 1, 1, 1] = [0, 1, 0, 0]$.

28. $\mathbf{u} + \mathbf{v} = [1, 1, 0, 1, 0] + [0, 1, 1, 1, 0] = [1, 0, 1, 0, 0]$.

29.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

30.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

·	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

31. $2 + 2 + 2 = 6 = 0$ in \mathbb{Z}_3 .

32. $2 \cdot 2 \cdot 2 = 3 \cdot 2 = 0$ in \mathbb{Z}_3 .

33. $2(2 + 1 + 2) = 2 \cdot 2 = 3 \cdot 1 + 1 = 1$ in \mathbb{Z}_3 .

34. $3 + 1 + 2 + 3 = 4 \cdot 2 + 1 = 1$ in \mathbb{Z}_4 .

35. $2 \cdot 3 \cdot 2 = 4 \cdot 3 + 0 = 0$ in \mathbb{Z}_4 .

36. $3(3 + 3 + 2) = 4 \cdot 6 + 0 = 0$ in \mathbb{Z}_4 .

37. $2 + 1 + 2 + 2 + 1 = 2$ in \mathbb{Z}_3 , $2 + 1 + 2 + 2 + 1 = 0$ in \mathbb{Z}_4 , $2 + 1 + 2 + 2 + 1 = 3$ in \mathbb{Z}_5 .

38. $(3 + 4)(3 + 2 + 4 + 2) = 2 \cdot 1 = 2$ in \mathbb{Z}_5 .

39. $8(6 + 4 + 3) = 8 \cdot 4 = 5$ in \mathbb{Z}_9 .

40. $2^{100} = (2^{10})^{10} = (1024)^{10} = 1^{10} = 1$ in \mathbb{Z}_{11} .

41. $[2, 1, 2] + [2, 0, 1] = [1, 1, 0]$ in \mathbb{Z}_3^3 .

42. $2[2, 2, 1] = [2 \cdot 2, 2 \cdot 2, 2 \cdot 1] = [1, 1, 2]$ in \mathbb{Z}_3^3 .

43. $2([3, 1, 1, 2] + [3, 3, 2, 1]) = 2[2, 0, 3, 3] = [2 \cdot 2, 2 \cdot 0, 2 \cdot 3, 2 \cdot 3] = [0, 0, 2, 2]$ in \mathbb{Z}_4^4 .
 $2([3, 1, 1, 2] + [3, 3, 2, 1]) = 2[1, 4, 3, 3] = [2 \cdot 1, 2 \cdot 4, 2 \cdot 3, 2 \cdot 3] = [2, 3, 1, 1]$ in \mathbb{Z}_5^4 .

44. $x = 2 + (-3) = 2 + 2 = 4$ in \mathbb{Z}_5 .

45. $x = 1 + (-5) = 1 + 1 = 2$ in \mathbb{Z}_6 .

46. $x = 2^{-1} = 2$ in \mathbb{Z}_3 .

47. No solution. 2 times anything is always even, so cannot leave a remainder of 1 when divided by 4.

48. $x = 2^{-1} = 3$ in \mathbb{Z}_5 .

49. $x = 3^{-1}4 = 2 \cdot 4 = 3$ in \mathbb{Z}_5 .

50. No solution. 3 times anything is always a multiple of 3, so it cannot leave a remainder of 4 when divided by 6 (which is also a multiple of 3).

51. No solution. 6 times anything is always even, so it cannot leave an odd number as a remainder when divided by 8.

52. $x = 8^{-1}9 = 7 \cdot 9 = 8$ in \mathbb{Z}_{11}
53. $x = 2^{-1}(2 + (-3)) = 3(2 + 2) = 2$ in \mathbb{Z}_5 .
54. No solution. This equation is the same as $4x = 2 - 5 = -3 = 3$ in \mathbb{Z}_6 . But 4 times anything is even, so it cannot leave a remainder of 3 when divided by 6 (which is also even).
55. Add 5 to both sides to get $6x = 6$, so that $x = 1$ or $x = 5$ (since $6 \cdot 1 = 6$ and $6 \cdot 5 = 30 = 6$ in \mathbb{Z}_8).
56. (a) All values. (b) All values. (c) All values.
57. (a) All $a \neq 0$ in \mathbb{Z}_5 have a solution because 5 is a prime number.
 (b) $a = 1$ and $a = 5$ because they have no common factors with 6 other than 1.
 (c) a and m can have no common factors other than 1; that is, the *greatest common divisor*, gcd, of a and m is 1.

1.2 Length and Angle: The Dot Product

- Following Example 1.15, $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = (-1) \cdot 3 + 2 \cdot 1 = -3 + 2 = -1$.
- Following Example 1.15, $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 6 \end{bmatrix} = 3 \cdot 4 + (-2) \cdot 6 = 12 - 12 = 0$.
- $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 1 = 2 + 6 + 3 = 11$.
- $\mathbf{u} \cdot \mathbf{v} = 3.2 \cdot 1.5 + (-0.6) \cdot 4.1 + (-1.4) \cdot (-0.2) = 4.8 - 2.46 + 0.28 = 2.62$.
- $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -\sqrt{2} \\ 0 \\ -5 \end{bmatrix} = 1 \cdot 4 + \sqrt{2} \cdot (-\sqrt{2}) + \sqrt{3} \cdot 0 + 0 \cdot (-5) = 4 - 2 = 2$.
- $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 1.12 \\ -3.25 \\ 2.07 \\ -1.83 \end{bmatrix} \cdot \begin{bmatrix} -2.29 \\ 1.72 \\ 4.33 \\ -1.54 \end{bmatrix} = -1.12 \cdot 2.29 - 3.25 \cdot 1.72 + 2.07 \cdot 4.33 - 1.83 \cdot (-1.54) = 3.6265$.
- Finding a unit vector \mathbf{v} in the same direction as a given vector \mathbf{u} is called *normalizing* the vector \mathbf{u} . Proceed as in Example 1.19:

$$\|\mathbf{u}\| = \sqrt{(-1)^2 + 2^2} = \sqrt{5},$$

so a unit vector \mathbf{v} in the same direction as \mathbf{u} is

$$\mathbf{v} = \frac{1}{\|\mathbf{u}\|} \mathbf{u} = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}.$$

- Proceed as in Example 1.19:

$$\|\mathbf{u}\| = \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13},$$

so a unit vector \mathbf{v} in the direction of \mathbf{u} is

$$\mathbf{v} = \frac{1}{\|\mathbf{u}\|} \mathbf{u} = \frac{1}{\sqrt{13}} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{13}} \\ -\frac{2}{\sqrt{13}} \end{bmatrix}.$$